Chapter 4

Three-phase full-wave AC voltage controllers

By
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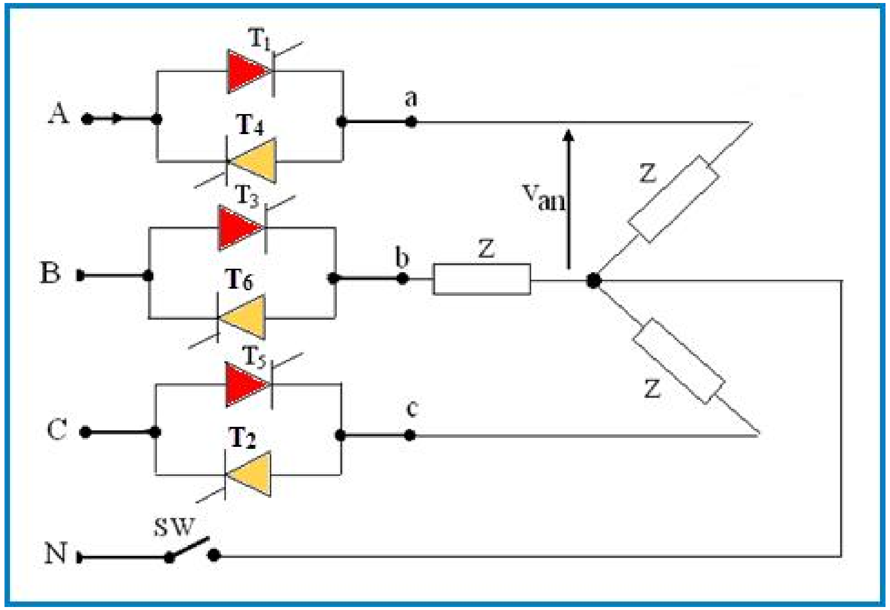
Three-phase full-wave AC voltage controllers with resistive load

Three-Phase Full-Wave AC Voltage Controller

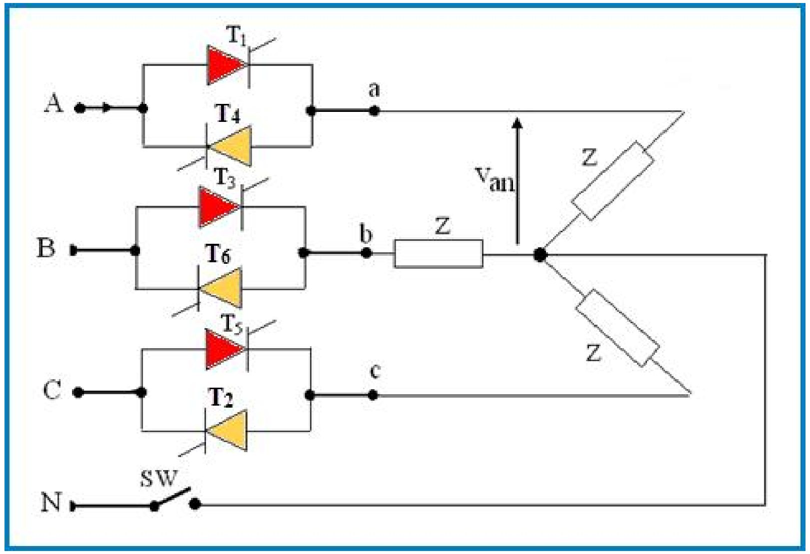
(Three-Wire, Star-Connected Load)

- The neutral of the supply is not connected to the neutral of the load.
- The firing sequance of thyristors is T_1, T_2, T_3, T_4, T_5, T_6.
- The gate-control circuit must be capable of triggering at the same instant two thyristor, one in each of two phases.
- These pulses are applied at intervals of 60° to various thyristors in a sequence the same as that of the supply voltage.



- The current flow to the load is controlled by the thyristors T_1, T_3 and T_5 and the thyristors T_2, T_4 and T_6 provide the return current path.

Three-Phase Full-Wave AC Voltage Controller



instantaneous input voltage per phase

V_S =RMS value of input ac supply

$$v_{AN} = \sqrt{2}\, V_s \sin \omega t$$

$$v_{BN} = \sqrt{2}\, V_s \sin(\omega t - 120°)$$

$$v_{CN} = \sqrt{2}\, V_s \sin(\omega t + 120°)$$

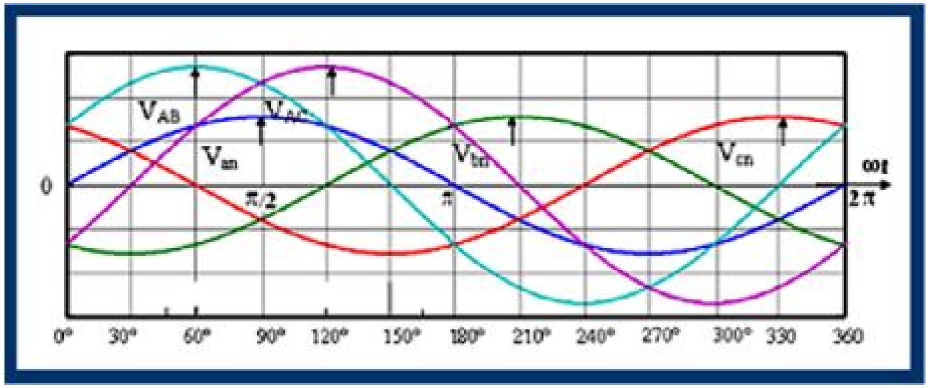
instantaneous input line voltages

$$v_{AB} = \sqrt{6}\, V_s \sin(\omega t + 30°)$$

$$v_{BC} = \sqrt{6}\, V_s \sin(\omega t - 90°)$$

$$v_{CA} = \sqrt{6}\, V_s \sin(\omega t + 150°)$$

Three-Phase Full-Wave AC Voltage Controller



- At any interval, either three SCRs or two SCRs, or no SCRs may be ON (conduct)

- If three SCRs conduct, a normal three-phase operation occurs and the output (load) phase voltage (v_{an}) is the same as the input (supply) phase voltage (v_{AN})

$$v_{an} = v_{AN} = \sqrt{2}\, V_s \sin \omega t$$

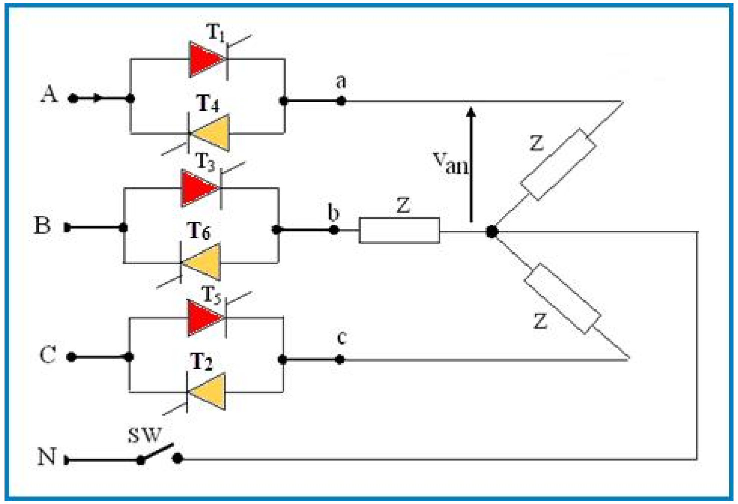
- If two SCRs conduct, the current flows only through two lines and the third line is open-circuited, and the output phase voltage (v_{an})

$$v_{an} = \frac{v_{AB}}{2} = \frac{\sqrt{3}\,\sqrt{2}\, V_s}{2} \sin\left(\omega t + \frac{\pi}{6}\right)$$

- If no SCRs conduct, the output phase voltage becomes zero.

$$v_{an} = 0$$

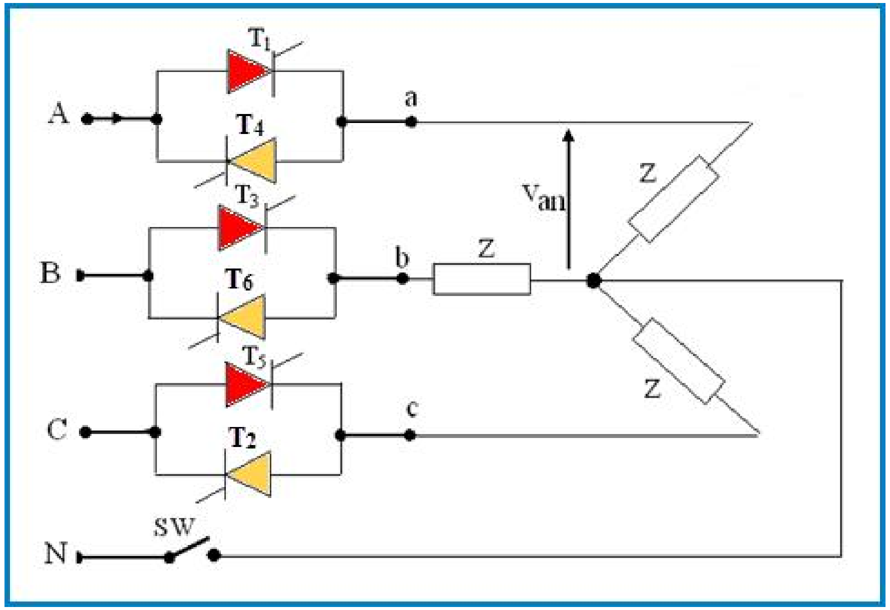
Three-Phase full-Wave AC Voltage Controller



- **Mode (0° £ α < 60°)** in this mode of operation, either two or three thyristors can conduct at the same time.
- **Mode (60° £ α < 90°)** in this mode of operation, at any time two thyristors, one in each phase, always conduct.
- **For (90° £ α < 150°)** in this mode of operation, at any time one or two thyristors conduct at the same time.

3-Phase full-Wave AC Voltage Controller Mode (0° £ α < 60°)

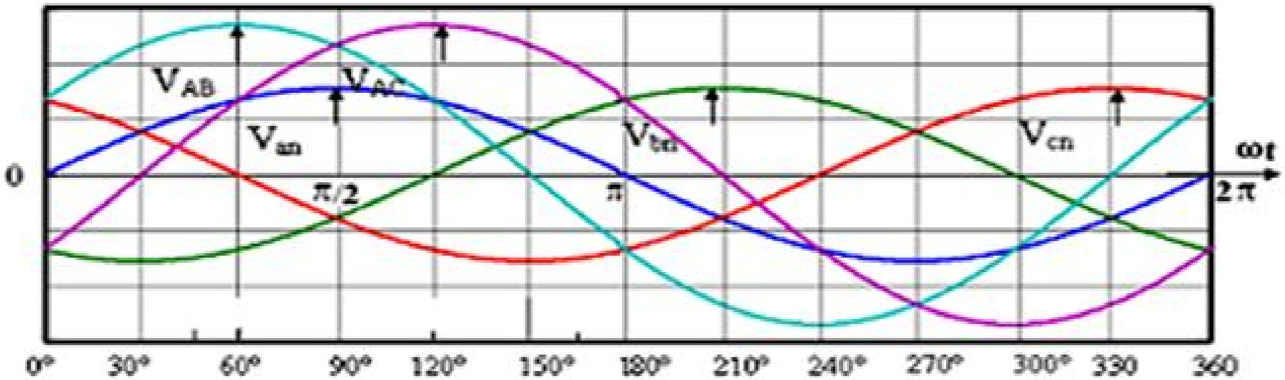
Switching Pattern of 3-Phase half-Wave AC Voltage Controller

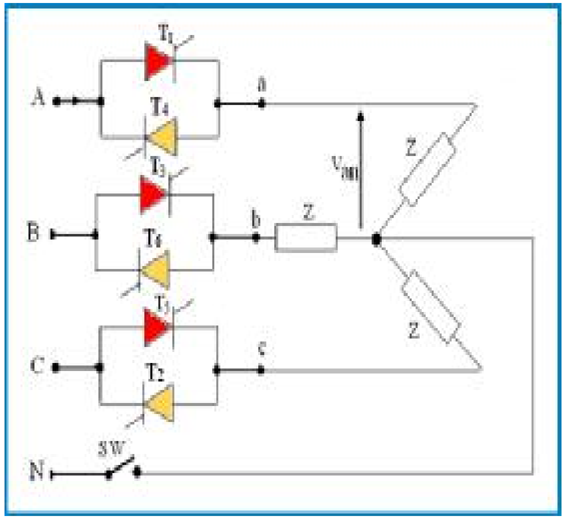


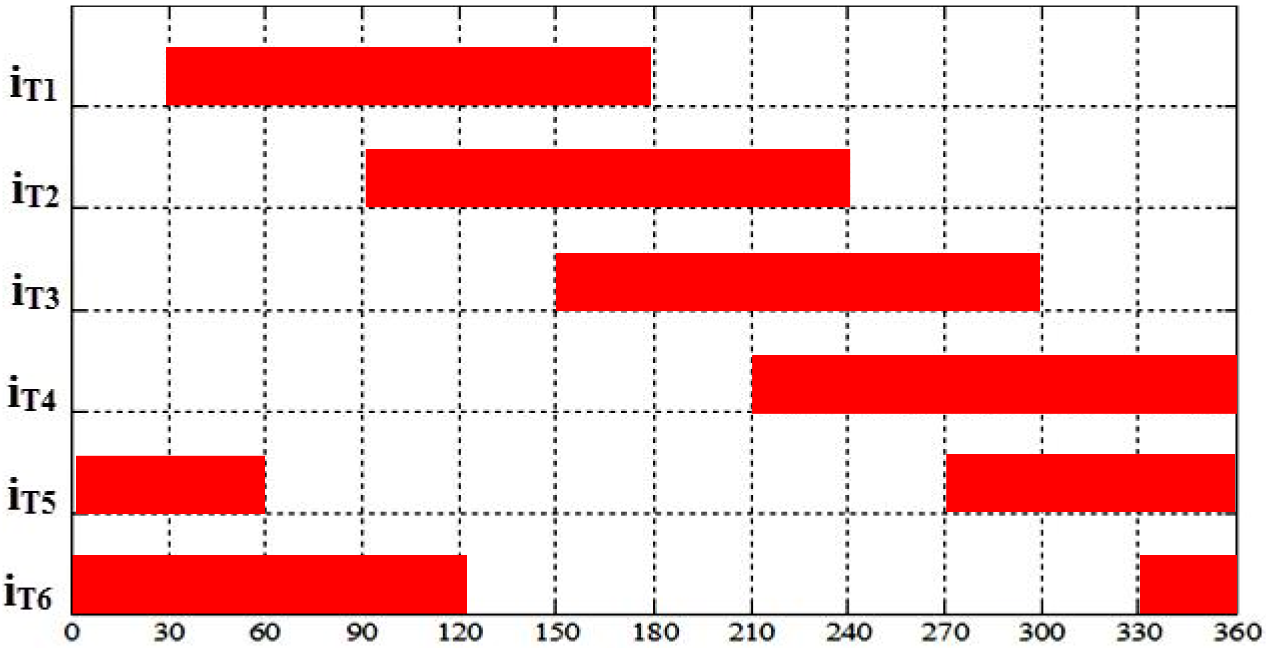
- **For (0° £ α < 60°)**

In this mode of operation, either two or three thyristors can conduct at the same time.

Switching Pattern of 3-Phase full-Wave AC Voltage Controller at (α = 30°)

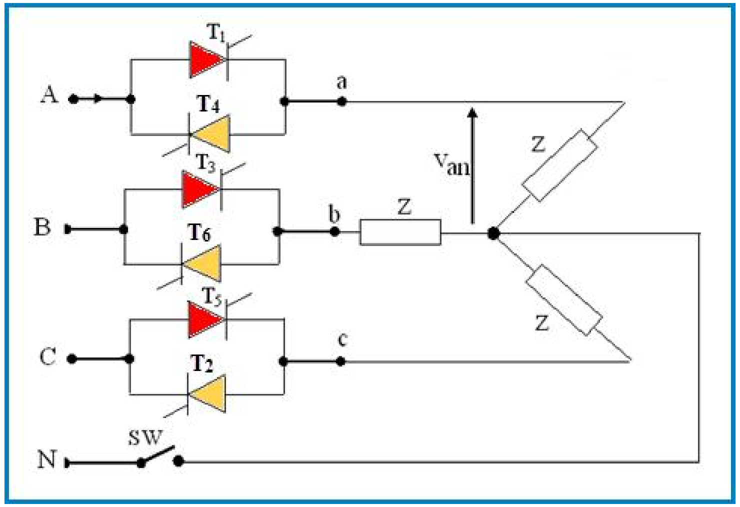


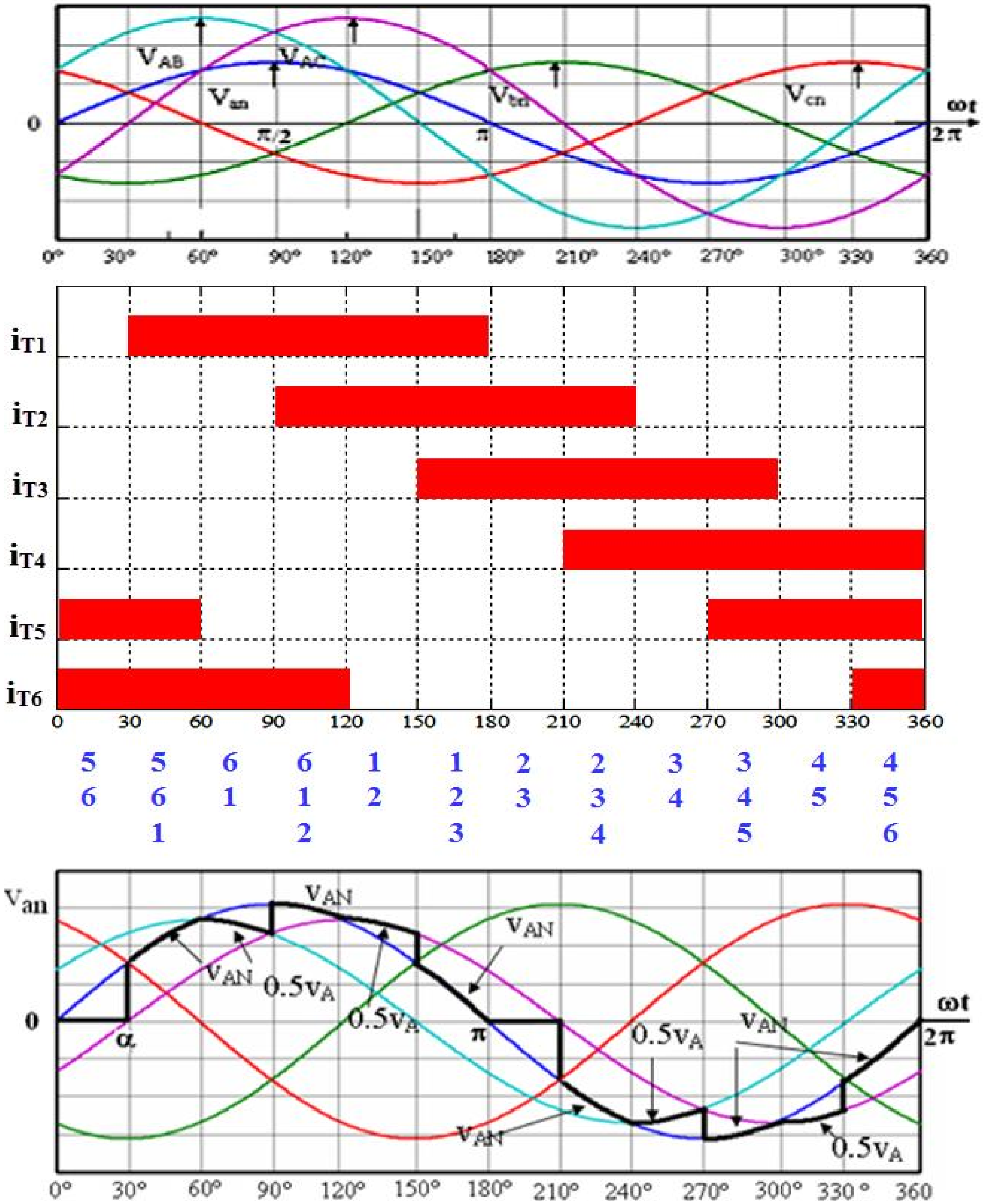




5	5	6	6	1	1	2	2	3	3	4	4
6	6	1	1	2	2	3	3	4	4	5	5
	1		2		3		4		5		6

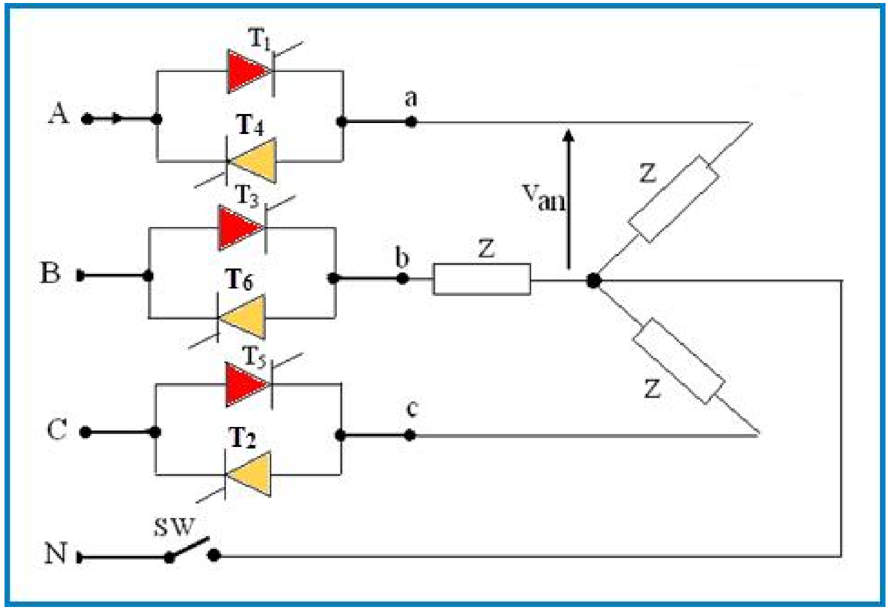
Output voltage waveform for three-phase, star-connected, three-wire full-wave AC voltage controller (α=30°).





3-Phase full-Wave AC Voltage Controller Mode (60° £ α < 90°)

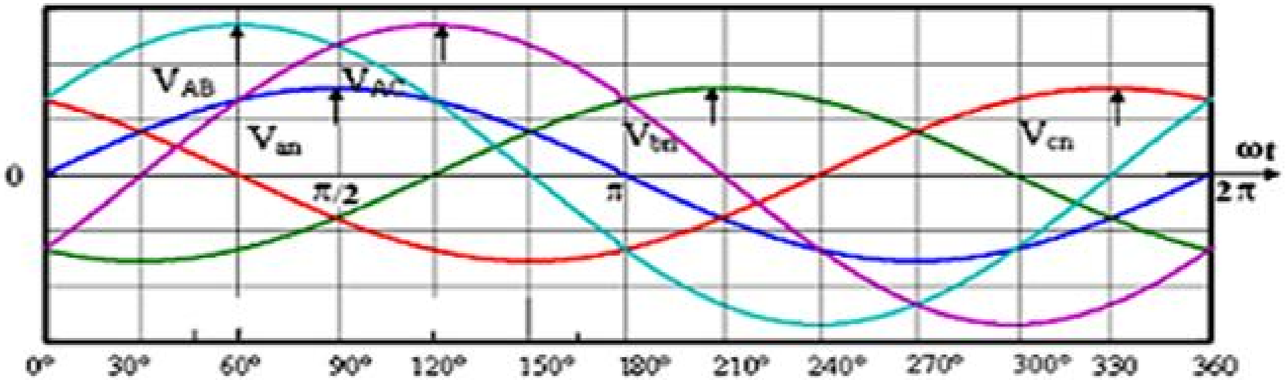
Switching Pattern of 3-Phase Half-Wave AC Voltage Controller

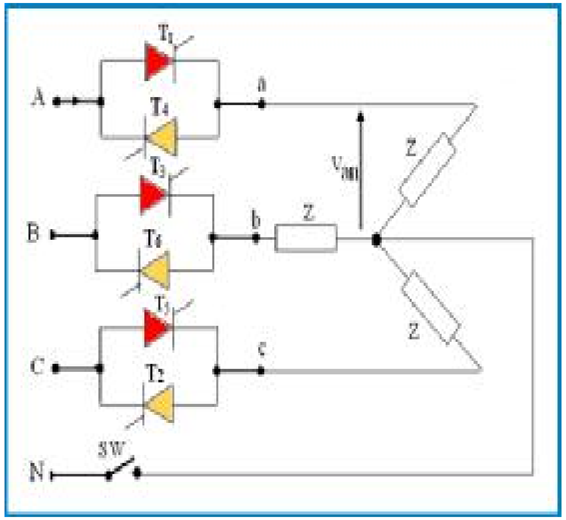


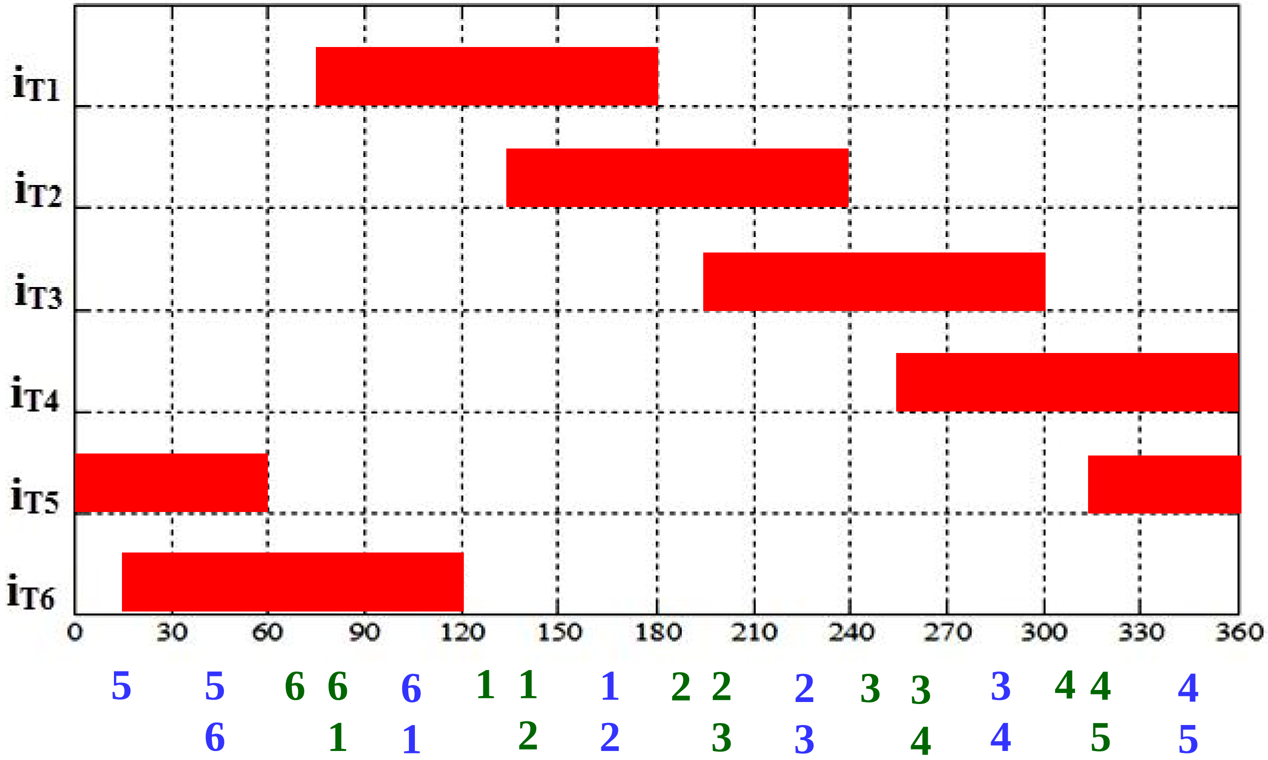
- **Mode (60° £ α < 90°)**

In this mode of operation, at any time two thyristors, one in each phase, always conduct.

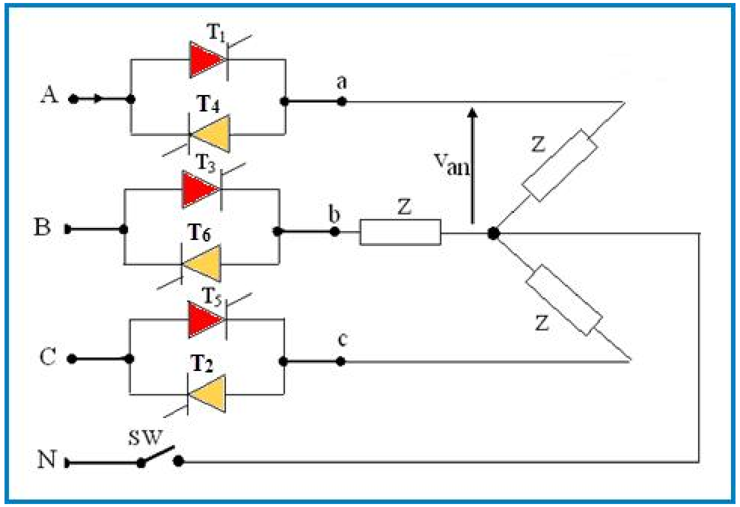
Switching Pattern of 3-Phase Full-Wave AC Voltage Controller at (α = 75°).

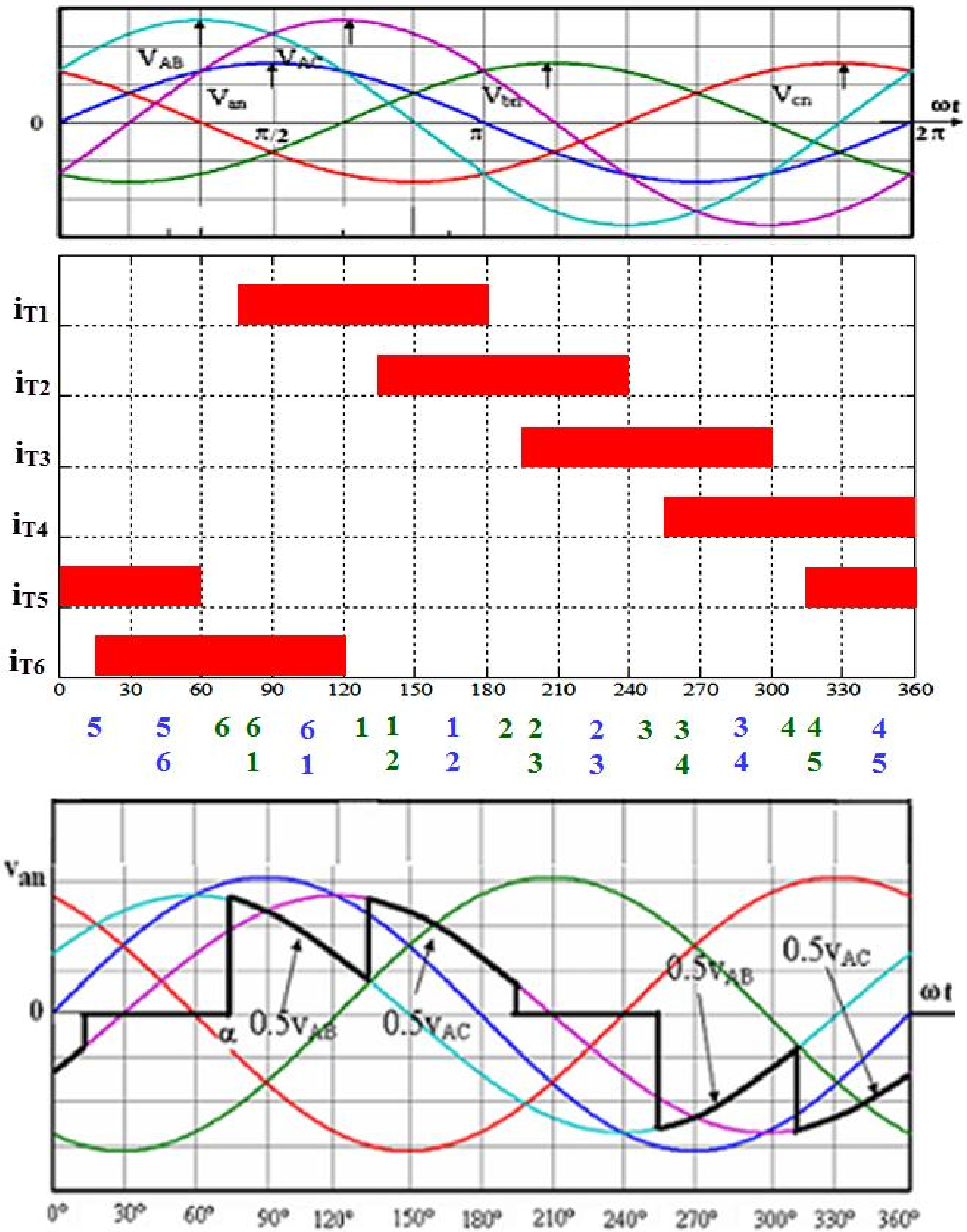






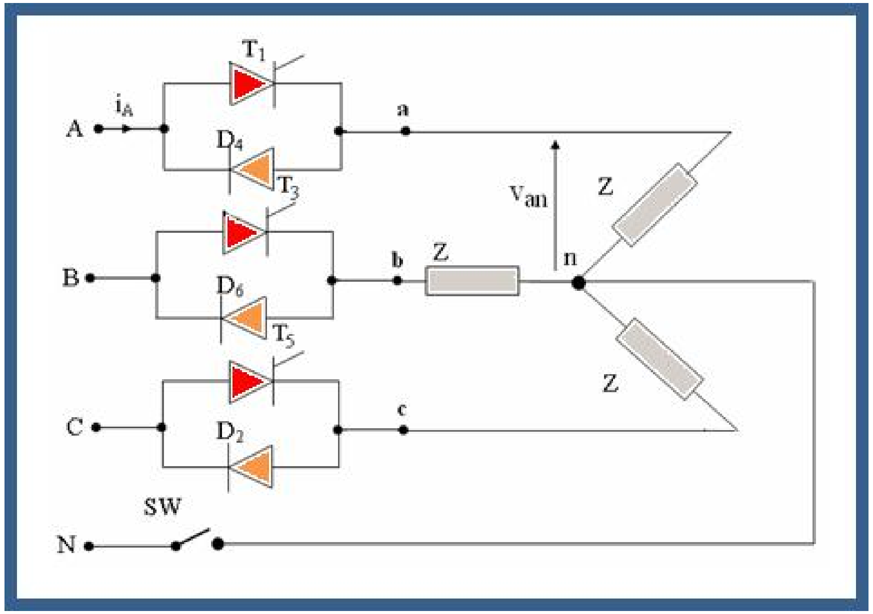
Output voltage waveform for three-phase, star-connected, three-wire full-wave AC voltage controller (α=75°).





3-Phase full-Wave AC Voltage Controller Mode $(90^\circ$ £ $\alpha < 150^\circ)$

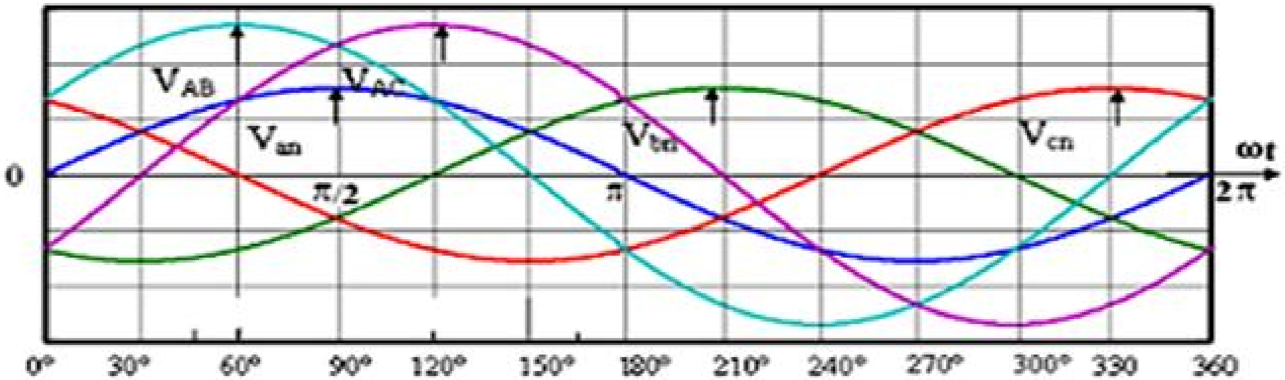
Switching Pattern of 3-Phase Half-Wave AC Voltage Controller

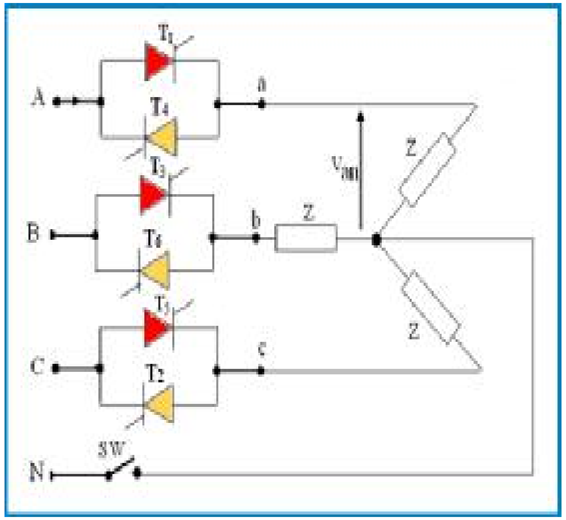


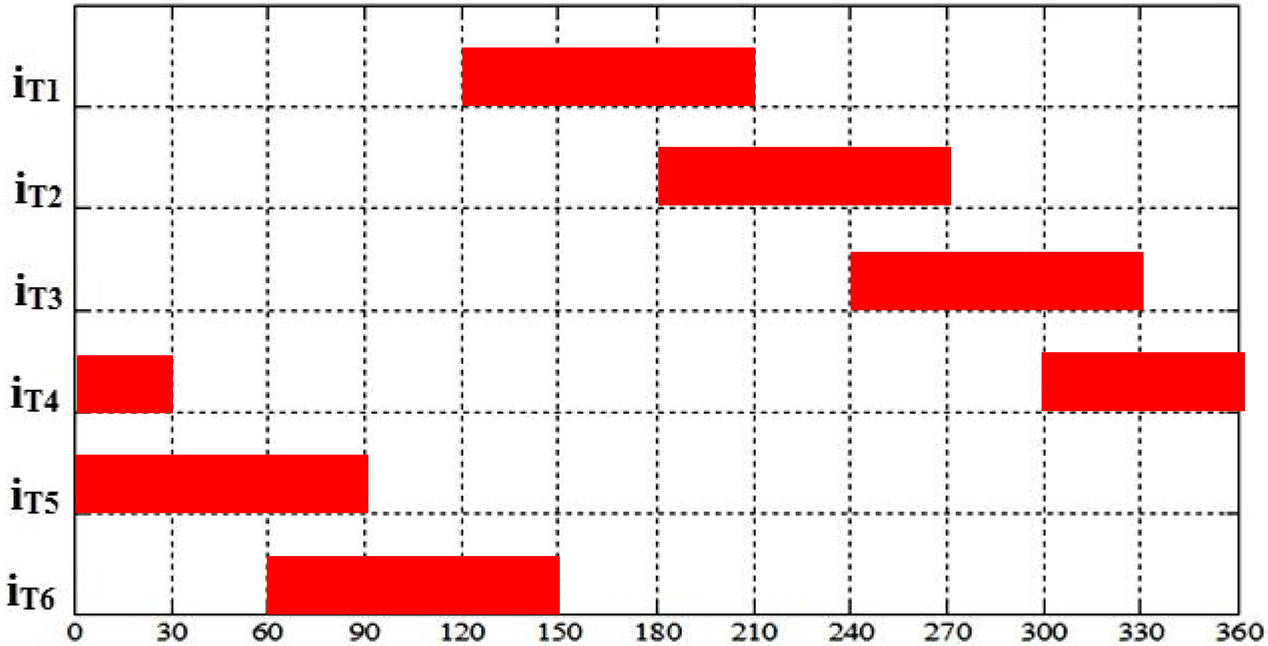
- **Mode (90° £ α < 150°)**

In this mode of operation, at any time one or two thyristors conduct at the same time.

Switching Pattern of 3-Phase Full-Wave AC Voltage Controller at (α = 120°).

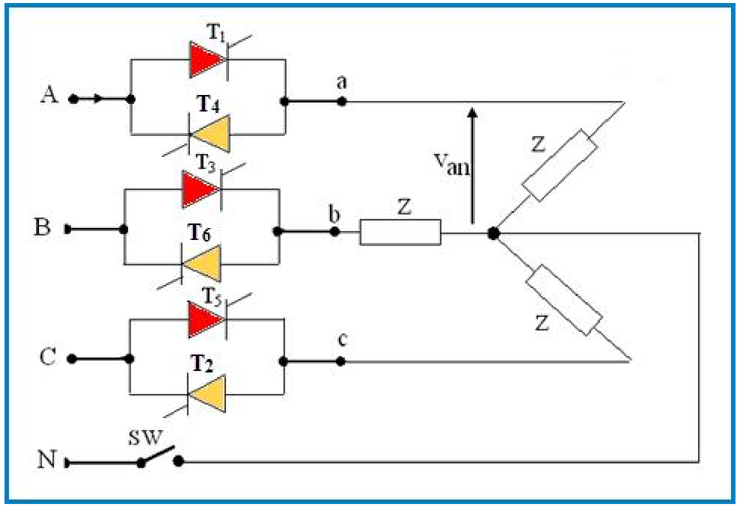


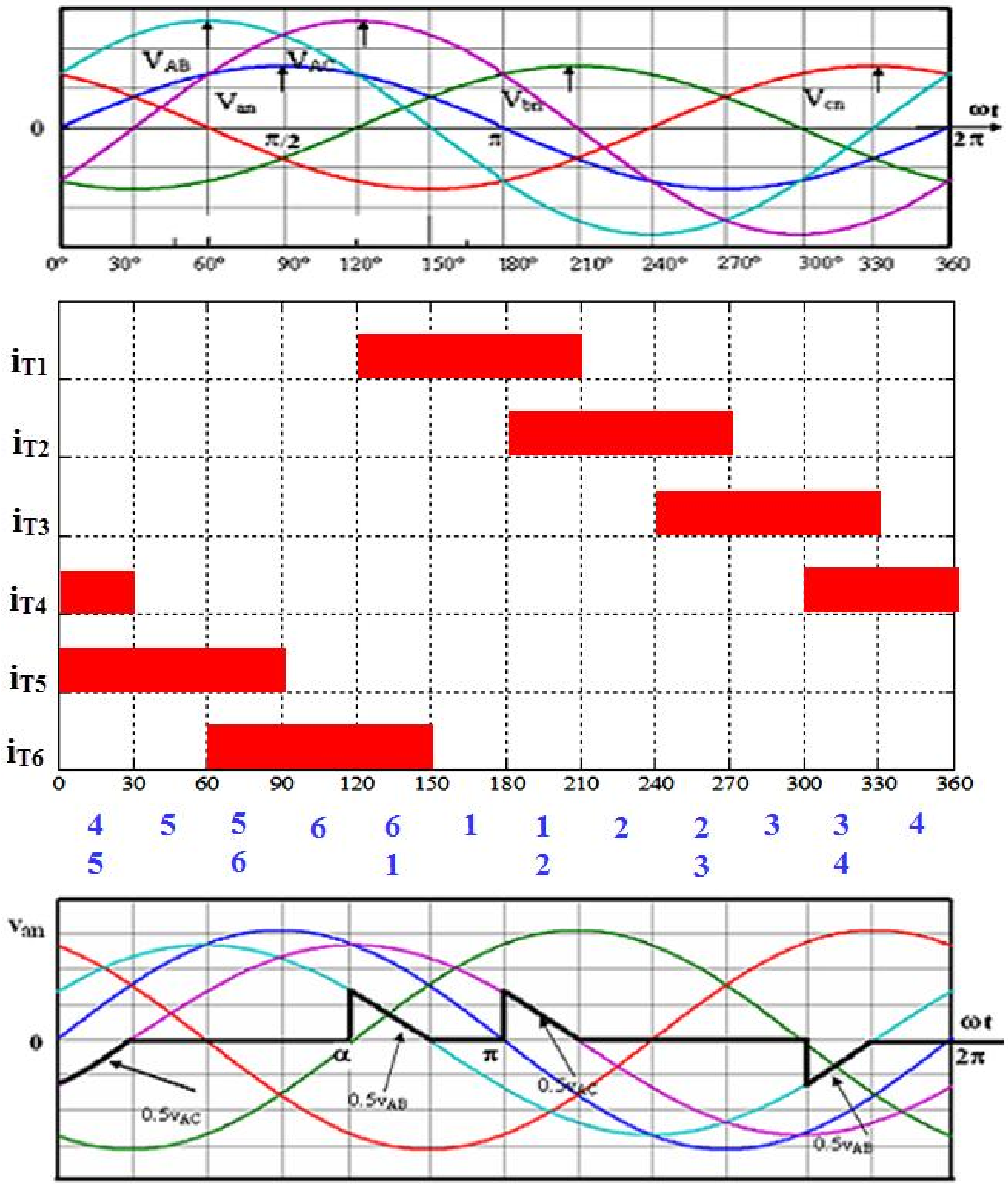




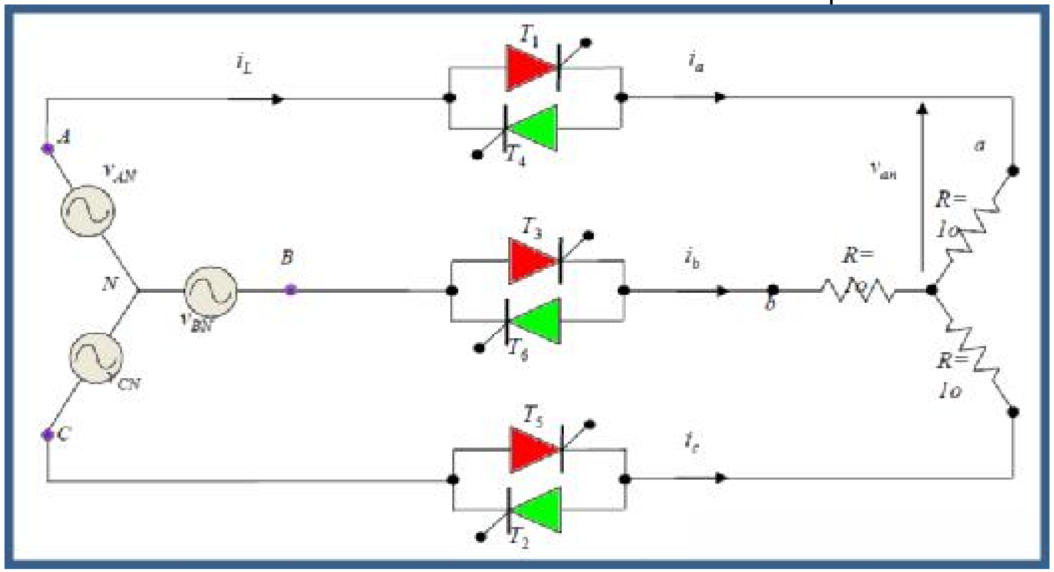
4	5	5	6	6	1	1	2	2	3	3	4
5		6		1		2		3		4	

Output voltage waveform for three-phase, star-connected, three-wire full-wave AC voltage controller (α=120°).





Analysis of three-phase Full-wave AC voltage controller with resistive load



RMS value of the output voltage

- The expressions of the rms value of the output voltage per phase for balanced star-connected resistive load are as follows:

$$V_0 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{an}^2 \, d(\omega t)}$$

$$v_{AN} = \sqrt{2}V_s \sin(\theta)$$

$$v_{BN} = \sqrt{2}V_s \sin\left(\theta - \frac{2\pi}{3}\right)$$

$$v_{BN} = \sqrt{2}V_s \sin\left(\theta + \frac{2\pi}{3}\right)$$

$$v_{AB} = \sqrt{6}V_s \sin\left(\theta + \frac{\pi}{6}\right)$$

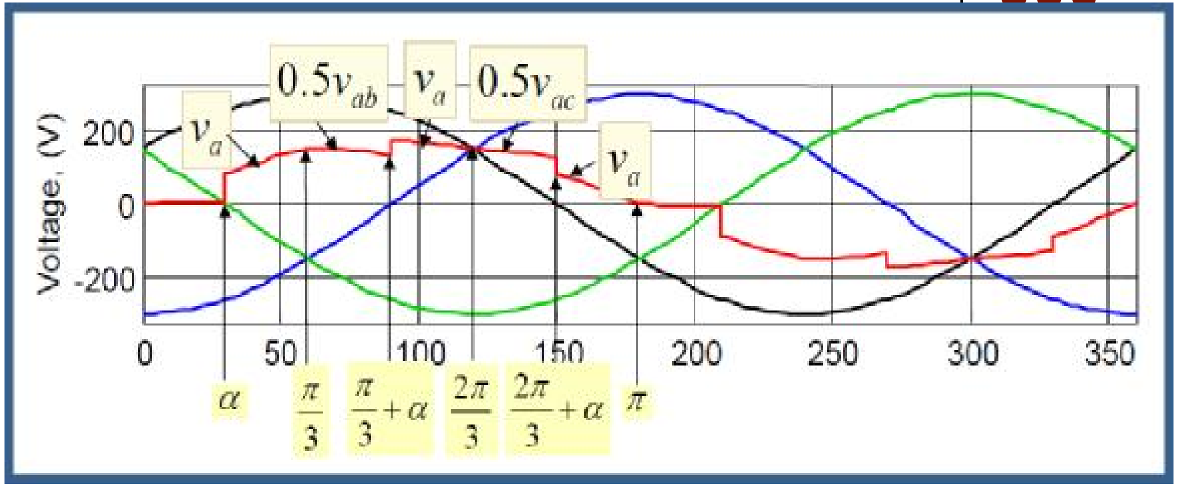
$$v_{BC} = \sqrt{6}V_s \sin\left(\theta - \frac{\pi}{2}\right)$$

$$v_{CA} = \sqrt{6}V_s \sin\left(\theta - \frac{\pi}{6}\right)$$

RMS output voltage at:

$$0 \le \alpha < 60^o$$

$$V_0 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{an}^2 d(\omega t)}$$



$$V_0 = \sqrt{\frac{1}{\pi}\left[\int_\alpha^{\pi/3} v_a^2 d(\omega t) + \int_{\pi/3}^{\pi/3+\alpha} \frac{v_{ab}^2}{4} d(\omega)t + \int_{\pi/3+\alpha}^{2\pi/3} v_a^2 d(\omega t) + \int_{2\pi/3}^{2\pi/3+\alpha} \frac{v_{ac}^2}{4} d(\omega t) + \int_{2\pi/3+\alpha}^{\pi} v_a^2 d(\omega t)\right]}$$

$$V_0 = \sqrt{6}V_s\sqrt{\frac{1}{\pi}\left[\int_\alpha^{\pi/3} \frac{\sin^2 \omega t}{3} d(\omega t) + \int_{\pi/3}^{\pi/3+\alpha} \frac{\sin^2(\omega t + \pi/6)}{4} d(\omega t) + \int_{\pi/3+\alpha}^{2\pi/3} \frac{\sin^2 \omega t}{3} d(\omega t) + \int_{2\pi/3}^{2\pi/3+\alpha} \frac{\sin^2(\omega t - \pi/6)}{4} d(\omega t) + \int_{2\pi/3+\alpha}^{\pi} \frac{\sin^2 \omega t}{3} d(\omega t)\right]}$$

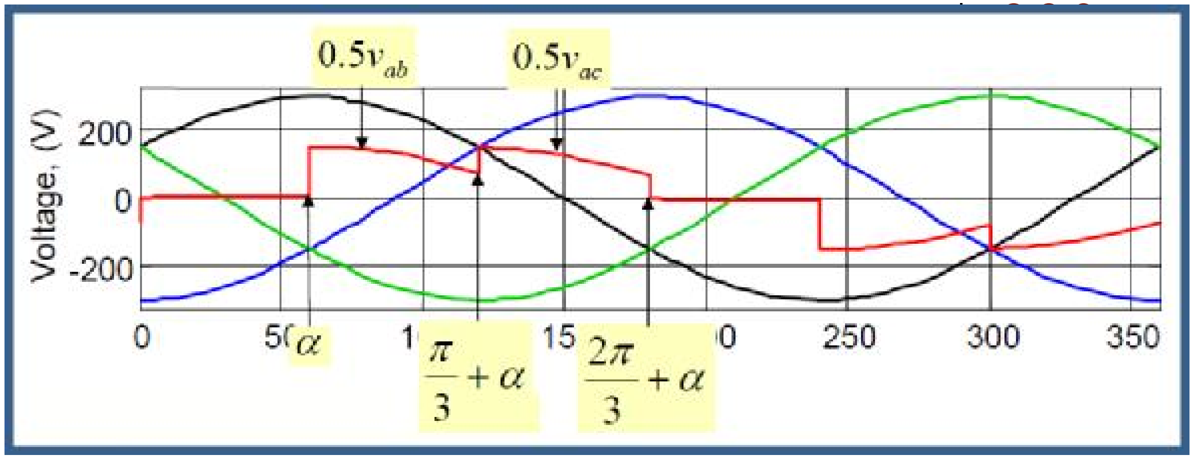
$$V_0 = \sqrt{6}V_s\sqrt{\frac{1}{\pi}\left(\frac{\pi}{6} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{8}\right)}$$

$$V_o = V_s\sqrt{\left\{1 - \frac{3\alpha}{2\pi} + \frac{3}{4\pi}\sin 2\alpha\right\}}$$

RMS output voltage at:

$$60^o \le \alpha < 90^o$$

$$V_0 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{an}^2 \, d(\omega t)}$$



$$V_0 = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\pi/3+\alpha} \frac{v_{ab}^2}{4} d(\omega t) + \int_{\pi/3+\alpha}^{2\pi/3+\alpha} \frac{v_{ac}^2}{4} d(\omega t)\right]}$$

$$V_0 = \sqrt{6}V_s \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{\pi/3+\alpha} \frac{\sin^2(\omega t + \pi/6)}{4} d(\omega t) + \int_{\pi/3+\alpha}^{2\pi/3+\alpha} \frac{\sin^2(\omega t - \pi/6)}{4} d(\omega t)\right]}$$

$$V_0 = \sqrt{6}V_s \sqrt{\frac{1}{\pi}\left[\int_{\alpha+\pi/6}^{\pi/3+\alpha+\pi/6} \frac{\sin^2 \omega t}{4} d(\omega t) + \int_{\pi/3+\alpha-\pi/6}^{2\pi/3+\alpha-\pi/6} \frac{\sin^2 \omega t}{4} d(\omega t)\right]}$$

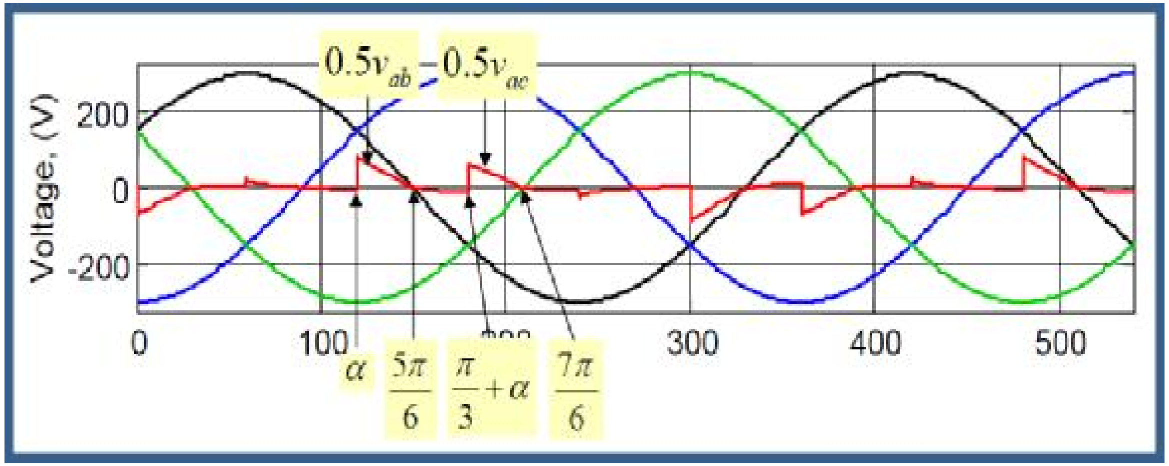
$$V_0 = \sqrt{6}V_s \sqrt{\frac{1}{\pi}\left[\int_{\alpha+\pi/6}^{\pi/2+\alpha} \frac{\sin^2 \omega t}{4} d(\omega t) + \int_{\pi/6+\alpha}^{\pi/2+\alpha} \frac{\sin^2 \omega t}{4} d(\omega t)\right]}$$

$$V_0 = \sqrt{6}V_s \sqrt{\frac{1}{\pi}\left(\frac{\pi}{12} + \frac{3\sin 2\alpha}{16} + \frac{\sqrt{3}\cos 2\alpha}{16}\right)}$$

$$V_o = V_s \sqrt{\left\{\frac{1}{2} + \frac{9}{8\pi}\sin 2\alpha + \frac{3\sqrt{3}}{8\pi}\cos 2\alpha\right\}}$$

RMS output voltage at:

$$90^o \leq \alpha < 150^o$$



$$V_0 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{an}^2 d(\omega t)}$$

$$V_0 = \sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{5\pi/6} \frac{v_{ab}^2}{4} d(\omega t) + \int_{\pi/3+\alpha}^{7\pi/6} \frac{v_{ac}^2}{4} d(\omega t)\right]}$$

$$V_0 = \sqrt{6}V_s\sqrt{\frac{1}{\pi}\left[\int_{\alpha}^{5\pi/6} \frac{\sin^2(\omega t + \pi/6)}{4} d(\omega t) + \int_{\pi/3+\alpha}^{7\pi/6} \frac{\sin^2(\omega t - \pi/6)}{4} d(\omega t)\right]}$$

$$V_0 = \sqrt{6}V_s\sqrt{\frac{1}{\pi}\left[\int_{\alpha+\pi/6}^{5\pi/6+\pi/6} \frac{\sin^2 \omega t}{4} d(\omega t) + \int_{\pi/3+\alpha-\pi/6}^{7\pi/6-\pi/6} \frac{\sin^2 \omega t}{4} d(\omega t)\right]}$$

$$V_0 = \sqrt{6}V_s\sqrt{\frac{1}{\pi}\left[\int_{\pi/6+\alpha}^{\pi} \frac{\sin^2 \omega t}{4} d(\omega t) + \int_{\pi/6+\alpha}^{\pi} \frac{\sin^2 \omega t}{4} d(\omega t)\right]}$$

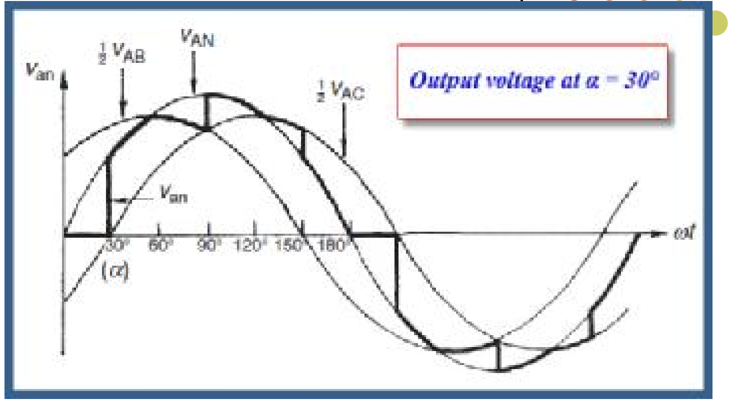
$$V_0 = \sqrt{6}V_s\sqrt{\frac{1}{\pi}\left(\frac{5\pi}{24} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{16} + \frac{\sqrt{3}\cos 2\alpha}{16}\right)}$$

$$V_o = V_s\sqrt{\left\{\frac{5}{4} - \frac{3\alpha}{2\pi} + \frac{3}{8\pi}\sin 2\alpha + \frac{3\sqrt{3}}{8\pi}\cos 2\alpha\right\}}$$

Analysis of three-phase Full-wave AC voltage controller with resistive load

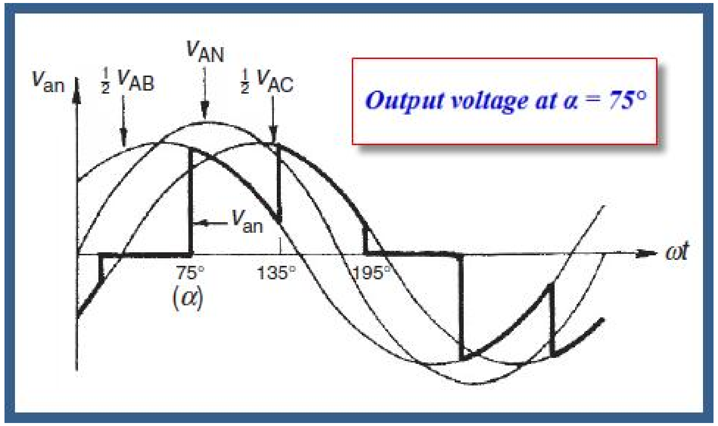
Mode I (also known as Mode 2/3): (0° £ α < 60°)

$$V_o = V_s \sqrt{\left\{ 1 - \frac{3\alpha}{2\pi} + \frac{3}{4\pi}\sin 2\alpha \right\}}$$



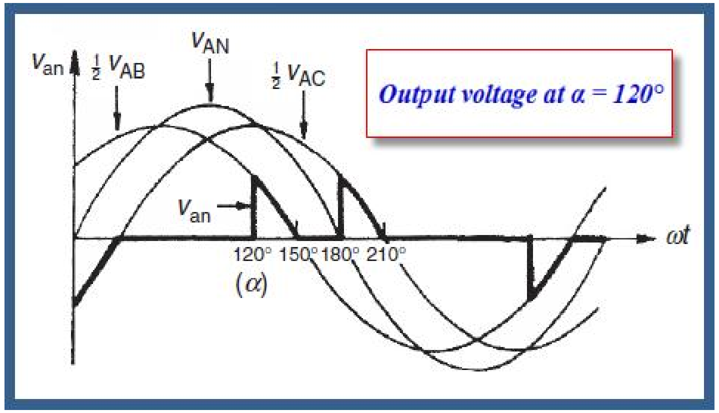
Mode II (also known as Mode 2/2): (60° £ α < 90°)

$$V_o = V_s \sqrt{\left\{ \frac{1}{2} + \frac{9}{8\pi}\sin 2\alpha + \frac{3\sqrt{3}}{8\pi}\cos 2\alpha \right\}}$$



Mode III (also known as Mode 0/2): (90° £ α < 150°)

$$V_o = V_s \sqrt{\left\{ \frac{5}{4} - \frac{3\alpha}{2\pi} + \frac{3}{8\pi}\sin 2\alpha + \frac{3\sqrt{3}}{8\pi}\cos 2\alpha \right\}}$$



Analysis of three-phase Full-wave AC voltage controller with resistive load

RMS Output current

$$I_0 = \frac{V_0}{R_L}$$

Where: $I_o = I_a$ is the Output (load) current in phase (a)

Output power

$$P_0 = 3 I_0^2 R_L$$

Input VA rating

$$VA = 3 V_s I_o$$

Supply power factor

$$PF = \frac{P_0}{VA}$$

Ex .1: **The three-phase bidirectional controller supplies a wye-connected resistive load of 10 Ω/phase and the line-to-line input voltage is 380-V, 50 Hz. The firing delay angle is 30°.**

(a) Draw the output phase voltage waveform.

(b) Drive the output phase voltage expression.

(b) Determine the rms output phase voltage and current.

(b) Determine the input power factor.

Solution

$V_L = 380v$ $f_s = 50$ Hz $R = 10$ ohm $\alpha = 30^o$

(a) the rms output phase voltage

Mode (1)

Firing angle range

(0° £ α < 60°)

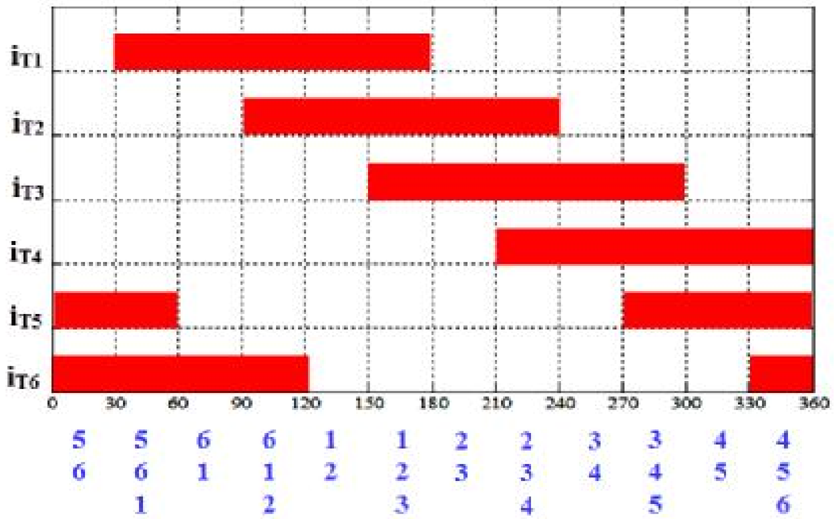
$$V_o = V_s\sqrt{\left\{1-\frac{3\alpha}{2\pi}+\frac{3}{4\pi}\sin 2\alpha\right\}}$$

$$V_s = 380/\sqrt{3} = 219.4v$$

$v_{AN} = \sqrt{2}\, V_s \sin \omega t$

$v_{AB} = \sqrt{6}\, V_s \sin(\omega t + \pi/6)$

$v_{AC} = \sqrt{6}\, V_s \sin(\omega t - \pi/6)$



The expression for the instantaneous output phase voltage, v_{an}, the positive half-cycle is:

$0 \le \omega t < \pi/6$: $v_{an} = 0$.

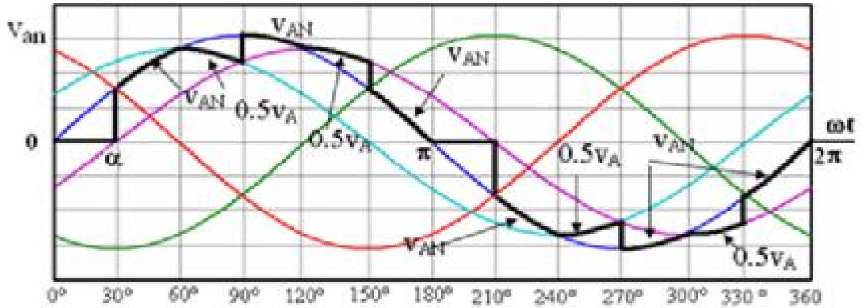
$\pi/6 \le \omega t < \pi/3$: $v_{an} = v_{AN} = \sqrt{2}\, V_s \sin(\omega t)$.

$\pi/3 \le \omega t < \pi/2$: $v_{an} = 0.5\, v_{AB} = \sqrt{6}/2\, V_s \sin(\omega t + \pi/6)$.

$\pi/2 \le \omega t < 2\pi/3$: $v_{an} = v_{AN} = \sqrt{2}\, V_s \sin(\omega t)$.

$2\pi/3 \le \omega t < 5\pi/6$: $v_{an} = 0.5\, v_{AC} = \sqrt{6}/2\, V_s \sin(\omega t - \pi/6)$.

$5\pi/6 \le \omega t < \pi$: $v_{an} = v_{AN} = \sqrt{2}\, V_s \sin(\omega t)$.



The rms value of the output phase voltage (V_o) can be derived as follows:

$V_{an(rms)}$

$$V_0 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{an}^2\, d(\omega t)}$$

$$V_o = \left\{ \frac{1}{\pi} \left[\int_{\pi/6}^{\pi/3} v_{AN}^2\, d\omega t + \int_{\pi/3}^{\pi/2} (0.5 v_{AB})^2\, d\omega t + \int_{\pi/2}^{2\pi/3} v_{AN}^2\, d\omega t + \int_{2\pi/3}^{5\pi/6} (0.5 v_{AC})^2\, d\omega t + \int_{5\pi/6}^{\pi} v_{AN}^2\, d\omega t \right] \right\}^{0.5}$$

$$V_o = \sqrt{6}\, V_s \left[\frac{1}{\pi} \left\{ \int_{\pi/6}^{\pi/3} \frac{1}{3}\sin^2\omega t.d\omega t + \int_{\pi/2}^{2\pi/3} \frac{1}{4}\sin^2\omega t.d\omega t + \int_{\pi/2}^{2\pi/3} \frac{1}{3}\sin^2\omega t.d\omega t + \int_{\pi/2}^{2\pi/3} \frac{1}{4}\sin^2\omega t.d\omega t + \int_{5\pi/6}^{\pi} \frac{1}{3}\sin^2\omega t.d\omega t \right\} \right]^{0.5}$$

$$V_s = 380/\sqrt{3} = 219.4v$$

$$V_o = V_s \sqrt{\left\{ 1 - \frac{3\alpha}{2\pi} + \frac{3}{4\pi}\sin 2\alpha \right\}}$$

$$V_o = V_s \left[1 - \frac{1}{4} + \frac{3}{4\pi}\sin\left(\frac{\pi}{3}\right) \right]^{0.5} = 215.2 \text{ V}$$

RMS Output current

$$I_0 = \frac{V_0}{R_L}$$

$$I_a = \frac{215.2}{10} = 21.52 \text{ A}$$

Output power

$$P_0 = 3 I_0^2 R_L$$

$$P_o = 3 \text{ x } (21.52)^2 \text{ X} 10 = 13.897 \text{ kW}$$

Input VA rating

$$VA = 3V_s\, I_o \implies VA = 3 \times 220 \times 21.52 = \boxed{14.203\ \text{kVA}}$$

Supply power factor

$$PF = \frac{P_0}{VA} \implies P.F. = \frac{13.893}{14.203} = \boxed{0.978}$$

Exercise:

(a) Repeat the previous example for α = 75° and 120°.

(b) Derive two general expressions for the rms output phase voltage for mode (II) and mode (III).